NASA TECHNICAL NOTE

NASA TN D-6250

C.1

NASA TN D-6250

LATERAL SLOSHING IN OBLATE SPHEROIDAL TANKS UNDER REDUCED- AND NORMAL-GRAVITY CONDITIONS

by Thom A. Coney and Jack A. Salzman

Lewis Research Center
Cleveland, Ohio 44135

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • MARCH 1971

1. Report No. NASA TN D-6250	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle LATERAL SLOSHING IN OBLATE SPHEROIDAL TANKS UNDER REDUCED- AND NORMAL-GRAVITY CONDITIONS		5. Report Date March 1971
		6. Performing Organization Code
7. Author(s) Thom A. Coney and Jack A. Salzman		8. Performing Organization Report No. E-5842
		10. Work Unit No. 124-08
9. Performing Organization Name and Address Lewis Research Center National Aeronautics and Space Administration Cleveland, Ohio 44135		11. Contract or Grant No.
		13. Type of Report and Period Covered Technical Note
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D.C. 20546		14. Sponsoring Agency Code
15. Supplementary Notes		

16. Abstract

An experiment was conducted to measure the natural lateral sloshing frequency of liquids in oblate spheroidal tanks. The eccentricities of the tanks used were 0, 0.68, and 0.8. The liquids chosen exhibited 0° static contact angles on the container walls. Tests were conducted under both reduced- and normal-gravity conditions. Resulting Bond numbers ranged from 5 to 927. The data were compared with low and high Bond number theory and with the results of previous studies.

17. Key Words (Suggested by Author(s)) Low gravity, Zero gravity, Sloshing, Low Bond number, High Bond number		18. Distribution Statement Unclassified - unlimited	
19. Security Classif. (of this report) Unclassified	20. Security Classif. (of this page) Unclassified	21. No. of Pages 19	22. Price* $3.00

*For sale by the National Technical Information Service, Springfield, Virginia 22151

LATERAL SLOSHING IN OBLATE SPHEROIDAL TANKS UNDER REDUCED- AND NORMAL-GRAVITY CONDITIONS

by Thom A. Coney and Jack A. Salzman

Lewis Research Center

SUMMARY

An experiment was conducted to measure the natural lateral sloshing frequency of liquids in oblate spheroidal tanks. The eccentricities of the tanks used were 0, 0.68, and 0.8. The liquids chosen exhibited 0° static contact angles on the container walls. Tests were conducted under both reduced- and normal-gravity conditions. Resulting Bond numbers ranged from 5 to 927. The data were compared with low and high Bond number theory and with the results of previous studies.

INTRODUCTION

With the development and use of high-energy liquid propellant space vehicles, there has grown an increased need to understand the general behavior of liquid-vapor systems in reduced- and normal-gravity environments. As a result, both theoretical and experimental analyses of this behavior have been undertaken. In one facet of this research, specific attention has been given to the dependence of the liquid-vapor interface shape and its sloshing characteristics on such system parameters as liquid properties, tank size and shape, and gravity level (ref. 1). Early studies were concerned primarily with spherical and cylindrical tank geometries. However, more recently, attention has been directed toward more complex geometries such as toroids and spheroids. Reduced-gravity studies of liquid behavior in oblate spheroidal tanks (refs. 2 and 3) have shown that the interface shape in these tanks varies markedly as a function of system parameters. (An oblate spheroid is formed by the rotation of an ellipse about its minor axis.) Attention is now focused on the reduced-gravity sloshing characteristics in these oblate spheroidal tanks.

Concus, Crane, and Satterlee (ref. 2) performed a theoretical analysis of small amplitude lateral sloshing for low Bond numbers in oblate spheroidal tanks for liquids exhibiting contact angles of 5°. This analysis showed that for oblate spheroidal tanks the natural frequency was a function of the defined Bond number, tank eccentricity, and filling. (The Bond number is defined using the tank semimajor axis as the characteristic length.) Calculations were made for fillings ranging from 12.5 to 87.5 percent, for Bond numbers ranging from 0 to 100 and for tank eccentricities of 0, 0.5, 0.68, and 0.80. Rattayya (ref. 4) analyzed sloshing in spheroids for those cases where capillary forces are negligible and the liquid-vapor interface is flat (i.e., high Bond number systems). References 2 and 4 thus provide analytical predictions which bracket the extremes of the Bond number regimes at varied fillings and tank eccentricities.

Normal-gravity experimental studies of sloshing in oblate spheroids have provided data for discrete portions of the Bond number range. References 5 and 6 considered only the high Bond number region, where the interface was essentially flat, while reference 7 investigated a range of Bond numbers from 55 to 172.

This report presents experimental data showing the variation of natural frequency with Bond number, tank shape, and filling for liquids in spheroids and compares these data to low and high Bond number theory (refs. 2 and 4). The experimental investigation reported here was conducted under both reduced- and normal-gravity conditions. The reduced-gravity data were obtained in a 5-second zero-gravity facility. Data were obtained for Bond numbers from 5 to 927, eccentricities from 0 to 0.8, and fillings from 25 to 87.5 percent. The data from this investigation and those published in reference 7 were compared with the theories of references 2 and 4.

SYMBOLS

a	system acceleration, cm/sec^2
B	Bond number, $\frac{ax^2}{\beta}$
e	eccentricity, $(1 - y^2/x^2)^{1/2}$
g	acceleration due to gravity, 981 cm/sec^2
$T_{1/2\ av}$	half period average, sec
x	semimajor axis (see fig. 4), cm
y	semiminor axis (see fig. 4), cm
β	specific surface tension, surface tension/density, cm^3/sec^2
Ω	natural frequency parameter, $\omega \Big/ \left(\frac{\beta}{x^3} + \frac{a}{x}\right)^{1/2}$
ω	natural frequency (lateral slosh), rad/sec

APPARATUS AND PROCEDURE

The reduced-gravity data presented in this report were obtained in the Lewis Zero Gravity Facility (fig. 1) by allowing the spheroidal tanks, contained in an experiment vehicle (fig. 2), to free fall the 142-meter depth of the vacuum chamber. Five seconds of free-fall time were realized in this manner. Evacuation of the chamber to a pressure of 13.3 newtons per square meter (1.3×10^{-4} atm) reduced system acceleration due to air drag to less than 10^{-5} g. The experiment was recovered in a cart filled with small pellets of expanded polystyrene. Average deceleration during recovery was 32 g's.

The experiment vehicle was a self-contained unit providing all the functions necessary for the experiment. The sloshing motion of the liquid-vapor interface was produced by a mechanical slide similar to that shown in figure 3. The sudden movement of the slide platform over a distance of less than 0.5 centimeter disturbed the interface sufficiently to produce the desired motion. The resulting motion was recorded by a high-speed (400 frame/sec) motion picture camera. Included in the field of view of the camera was a digital clock accurate to 0.01 second. A cold gas thrust system (fig. 2) provided accelerations other than zero gravity. Accelerations used ranged from 1.89×10^{-2} to 3.20×10^{-2} g. A more detailed discussion of the facility and the experiment vehicle can be found in reference 8.

Normal-Gravity Tests

The normal-gravity data were obtained using the apparatus shown in figure 3. This apparatus consisted of a slide platform on which the spheroidal tanks were placed, a digital clock accurate to 0.01 second, and a dc motor which imparted motion to the platform. The desired lateral sloshing of the liquid in the spheroidal tanks was produced as described previously by moving the platform a short distance with a single pulse from the dc motor. Again, a high-speed camera was used to record the liquid motion.

Test Containers and Liquids

Test containers were oblate spheroids formed from clear acrylic plastic. Values of eccentricity e of these spheroidal tanks were 0, 0.68, and 0.8. Three tank sizes were used with semimajor axes of 2, 3, and 4 centimeters. Carbon tetrachloride, ethanol, FC-78, and Freon-TF were used as test liquids. Surface tensions, densities, and viscosities for these liquids are presented in table I. Low-viscosity liquids were used to minimize damping and dynamic contact angle effects on the natural frequency. The carbon tetrachloride and ethanol were analytic reagent grade; the fluorocarbons were precision cleaning grade. All liquids exhibited 0° static contact angles on the spheroid walls. A small amount of dye was added to the liquids to improve the quality of the photography. The dye had no measurable effect on the pertinent liquid properties.

To ensure that the liquid and tank wall properties were not affected by contaminants, the spheroidal tanks were cleaned and filled in a class 10 000 clean room. The tanks were cleaned ultrasonically in a detergent-water solution, rinsed with water and methanol, and dried in a warm air dryer. Prior to each test, the tanks were rinsed with the test liquid, filled, and sealed.

RESULTS AND DISCUSSION

As mentioned previously, this report presents experimental data showing the variation of natural frequency with Bond number, tank shape, and filling for liquids in spheroids and compares these data to low and high Bond number theory (refs. 2 and 4). By appropriate use of tank size, liquid properties, and acceleration level, Bond numbers ranging from 5 to 927 were obtained providing data extending from the low Bond number region well into the high Bond number region.

Data Reduction

The natural frequency can be expressed in the dimensionless form (ref. 2)

$$\Omega = \omega \Bigg/ \left(\frac{\beta}{x^3} + \frac{a}{x}\right)^{1/2}$$

where ω is the natural frequency (lateral slosh) of the liquid in radians per second, x is the tank semimajor axis, β is the specific surface tension, and a is the system acceleration.

The natural frequency ω was determined by measuring and plotting the displacement of the liquid-vapor interface as a function of time. A film analyzer was used to facilitate this measurement. Because of refraction, especially for the high-eccentricity tanks, the behavior of the interface at and near the tank wall could not be considered. Instead, measurements were made on that portion of the interface that was most nearly flat (fig. 4), where displacements normal to the interface were most easily observed. Figure 5 is a plot showing interface oscillations for a normal-gravity test. The two sinusoidal curves represent the normal displacement of points on the interface equidistant from the tank centerline. The natural frequency was calculated from these plots by determining the half-period arithmetic average; that is,

$$\omega = \frac{\pi}{T_{1/2\ av}}$$

The normal-gravity tests provided a large number of half-period samples (e.g., fig. 5); however, time limitations restricted the low-gravity data to only one or two slosh oscillations per test. Because of the longer half-period times involved, these low-gravity measurements could be made with increased accuracies. Since damping was small, this measured frequency was assumed to be the natural frequency.

In all cases, the frequency measurement was made for the fundamental lateral sloshing mode. The lateral impulse to the tank usually excited higher modes of oscillation or surface perturbations; however, these damped away quickly. Figure 6 shows the fundamental mode for representative Bond numbers ranging from 5 to 927 at a filling of 25 percent and an eccentricity of 0.8. As would be expected for data encompassing both the high and low Bond number regions, the interface shape varied from flat to highly curved. Figure 7 shows the mode shape dependence on eccentricity for a filling of 25 percent and a Bond number of 10.

Comparison of Experimental and Theoretical Results

The data obtained in this experiment for both the reduced- and normal-gravity cases are presented in tables II and III, respectively, and in figures 8 to 10. For comparison with these data, curves representing the low and high Bond number theories are included in these figures. Before discussing the results, it is important to note certain limitations associated with these two theories. The theoretical solutions to both the low and high Bond number problems were obtained numerically. Consequently, closed-form solutions are not available, and only those particular eccentricities and fillings presented in references 2 and 4 can be considered. Reference 2 gives discrete data points for specific Bond numbers, eccentricities, and fillings but gives no data for Bond numbers greater than 100. Reference 4 gives data only for eccentricities of 0 and 0.86 rather than 0 and 0.8.

The comparisons presented in figures 8 to 10 clearly show the dependence of the natural sloshing frequency on Bond number, tank eccentricity, and filling. For example, the information in these figures shows that the greatest percentage change in natural frequency for a change in eccentricity results for the low Bond number, low-filling cases;

the greatest percentage change for a change in filling results for the low Bond number, high-eccentricity cases.

Figures 8 to 10 show the good agreement between the experimental data and theory. In the Bond number region below 30, the low Bond number theory of reference 2 applies very well. In the region above $B = 500$, the high Bond number theory of reference 4 applies very well. However, in the region between $B = 30$ and 500, the theory (high or low) that is applicable is dependent on the particular tank eccentricity and filling being used. That is, the transition point between the designated "high" and "low" Bond number regions varies with tank eccentricity and filling. (This is shown more clearly in figs. 11 and 12, which include data from ref. 7.) For example, for an eccentricity of 0.8 and a filling of 75 percent, the transition point is around $B = 400$, and for an eccentricity of 0 and a filling of 25 percent, this point is around $B = 70$.

This variation with eccentricity and filling of the transition point between the high and low Bond number regions is related to the curvature of the liquid-vapor interface shape. High Bond number theory is formulated assuming a flat interface with a 90° contact angle, while low Bond number theory assumes a number of curved interface shapes at near 0° contact angles. Consequently, it is reasonable to expect those combinations of Bond number, tank eccentricity, and filling which result in flat interfaces to satisfy high Bond number theory and those combinations which result in curved interfaces to satisfy low Bond number theory. This is illustrated in figure 13. Figure 13(a) shows the liquid-vapor interface shape for a filling of 75 percent with a Bond number of 927 and a filling of 25 percent with a Bond number of 139. (The eccentricity is 0.8 for all data shown in fig. 13.) The interface for both cases is flat and, as expected, their natural frequency is predicted by high Bond number theory. Figure 13(b) shows the interface shape for a filling of 75 percent and a Bond number of 139 and for a filling of 25 percent and a Bond number of 30. Even though the Bond number value of 139 appeared previously in the high Bond number region with a flat interface, neither liquid-vapor interface shown in figure 13(b) is flat. (The primary curvature in the $B = 139$ case in fig. 13(b) occurs near the tank wall.) The natural frequency in both cases is predicted by low Bond number theory. Thus, it is tank eccentricity and filling as well as Bond number that determine the regions of applicability of either the high or low Bond number theories.

SUMMARY OF RESULTS

An experimental investigation was conducted to measure the natural frequency of lateral sloshing in oblate spheroids. The semimajor axes of the spheroids used were 2, 3, and 4 centimeters with eccentricities of 0, 0.68, and 0.8. Bond numbers based on the tank semimajor axis ranged from 5 to 927. Tank fillings ranged from 25 to

87.5 percent. Test liquids were restricted to those which possessed near 0° static contact angles on the spheroid surfaces. The viscosities of these liquids ranged from 0.70 to 1.20 centipoise; consequently, viscous effects such as dynamic variations in contact angles were negligible. The surface tensions ranged from 13.2 to 26.9 dynes per centimeter (13.2×10^{-5} to 26.9×10^{-5} N/cm), and the densities ranged from 0.79 to 1.73 grams per cubic centimeter. Experiments were conducted in both reduced- and normal-gravity environments. The study yielded the following results:

1. The greatest percentage change in natural frequency for a chnage in eccentricity results when the Bond number and filling are low. The greatest percentage change for a change in filling results when the Bond number is low and the eccentricity is high.

2. The measured natural frequency compared well with that predicted by Concus, Crane, and Satterlee and Rattayya.

3. The transition point between the regions where high or low Bond number theory is applicable is a function of Bond number, tank eccentricity, and filling.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, December 3, 1970,
124-08.

REFERENCES

1. Abramson, H. Norman: The Dynamic Behavior of Liquids in Moving Containers, With Applications to Space Vehicle Technology. NASA SP-106, 1966.

2. Concus, P.; Crane, G. E.; and Satterlee, H. M.: Small Amplitude Lateral Sloshing in Spheroidal Containers Under Low Gravitational Conditions. Rep. LMSC/A944673, Lockheed Missiles and Space Co. (NASA CR-72500), Feb. 4, 1969.

3. Salzman, Jack A.: Low-Gravity Liquid-Vapor Interface Configurations in Spheroidal Containers. NASA TN D-5648, 1970.

4. Rattayya, Jasti V.: Sloshing of Liquids in Axisymmetric Ellipsoidal Tanks. Paper No. 65-114, AIAA, Jan. 1965.

5. Stofan, Andrew J.: Comparison of Propellant Sloshing Parameters Obtained from Model and Full-Size Centaur Liquid-Oxygen Tanks. NASA TM X-1286, 1966.

6. Leonard, Wayne H.; and Walton, William C., Jr.: An Investigation of the Natural Frequencies and Mode Shapes of Liquids in Oblate Spheroidal Tanks. NASA TN D-904, 1961.

7. Dodge, Franklin T.; and Garza, Luis R.: Slosh Force, Natural Frequency, and Damping of Low-Gravity Sloshing in Oblate Ellipsoidal Tanks. Tech. Rep. 7, Southwest Research Inst. (NASA CR-98443), Feb. 1969.

8. Salzman, Jack A.; and Masica, William J.: Lateral Sloshing in Cylinders Under Low-Gravity Conditions. NASA TN D-5958, 1969.

TABLE I. - SUMMARY OF LIQUID PROPERTIES

Liquid (at 20° C)	Surface tension, dynes/cm (10^{-5} N/cm)	Density, g/cm^3	Viscosity, cP
Carbon tetrachloride	26.9	1.59	0.97
Ethanol	22.3	.79	1.20
FC-78[a]	13.2	1.73	.82
Freon-TF[b]	18.6	1.58	.70

[a]A fluorocarbon solvent obtained from Minnesota Mining and Manufacturing Co.

[b]Trichlorotrifluoroethane obtained from E. I. Dupont de Nemours and Co.

TABLE II. - SUMMARY OF LOW-GRAVITY DATA

Liquid	Semimajor axis, x, cm	Eccentricity, e	System acceleration, a, cm/sec^2	Bond number, B	Filling, percent	Measured natural frequency, ω, rad/sec
Carbon tetrachloride	4	0.68	31.4	30	25	2.88
					75	3.38
	2	0.8	20.6	5	25	1.75
Ethanol	3	0.8	31.4	10	25	2.00
		0	31.4	10	25	3.49
					75	3.98
FC-78	3	0.8	25.5	30	25	2.24
					50	2.62
					75	3.17
		0	25.5	30	25	3.36
					50	3.61
					75	3.70
	2	0.68	18.6	10	50	3.17

TABLE III. - SUMMARY OF NORMAL-GRAVITY DATA

Liquid	Semimajor axis, x, cm	Eccentricity, e	Bond number, B	Filling, percent	Measured natural frequency, ω, rad sec
Carbon tetrachloride	4	0.8	927	25	14.8
				50	17.4
				75	20.6
				87.5	23.3
		0.68	927	25	15.8
				50	18.2
				75	21.1
				87.5	24.0
Ethanol	2	0.8	139	25	20.3
				50	23.1
				75	26.9
				87.5	28.8
		0.68	139	87.5	30.8
FC-78	2	0.8	514	25	20.9
				50	24.0
				75	29.1
				87.5	32.4
		0.68	514	25	22.4
				50	25.5
				75	29.9
				87.5	33.2
Freon-TF	2	0.8	333	75	28.3
		.8	333	87.5	30.8
		.68	333	87.5	32.1
		0	333	50	27.3

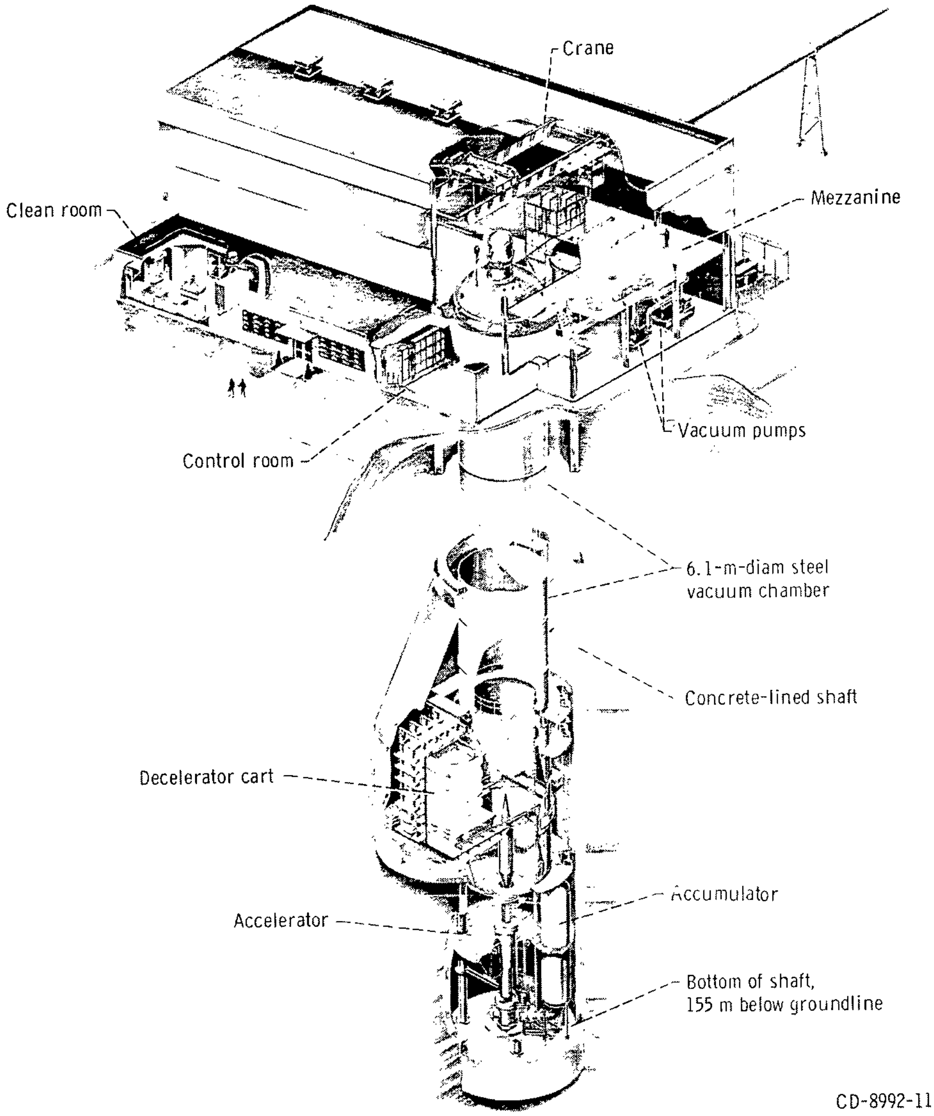


Figure 1. - Zero-gravity research facility.

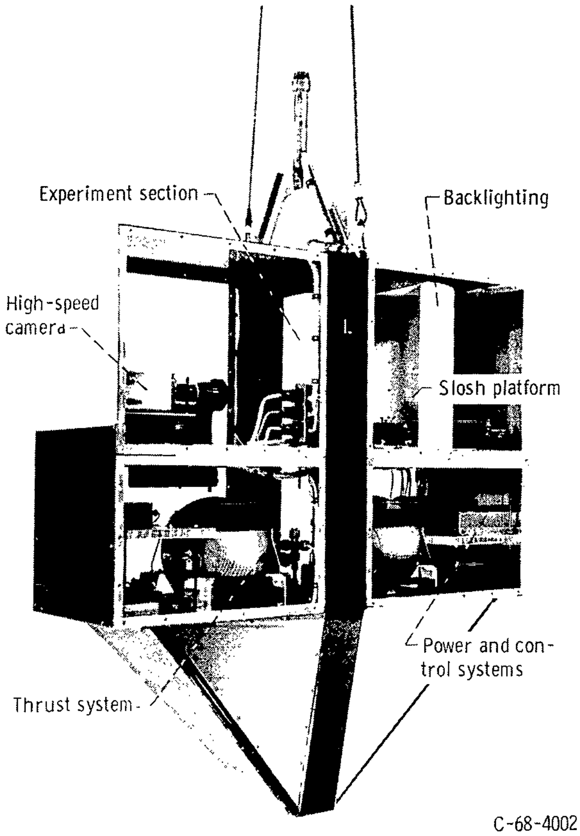


Figure 2. - Experiment vehicle for low-gravity tests.

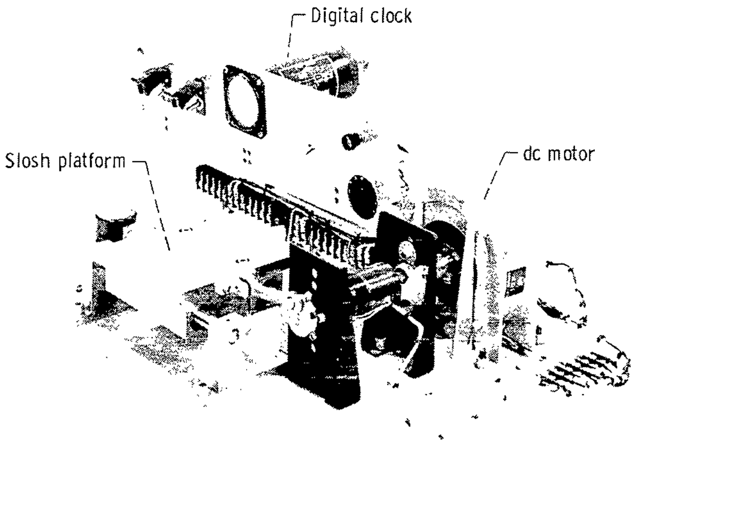
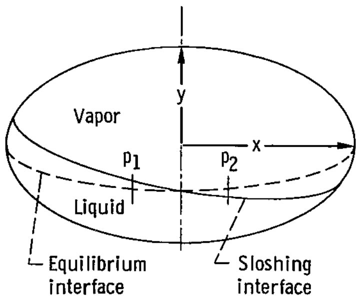


Figure 3. - Apparatus for normal-gravity tests.



Interface displacement measurements made at points such as p_1 and p_2

Figure 4. - Test geometry.

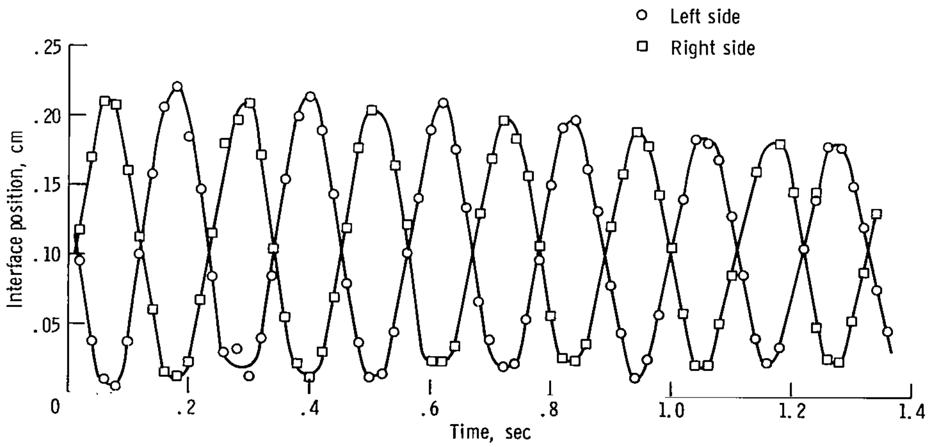


Figure 5. - Sample data plot of lateral slosh. Bond number, 514.

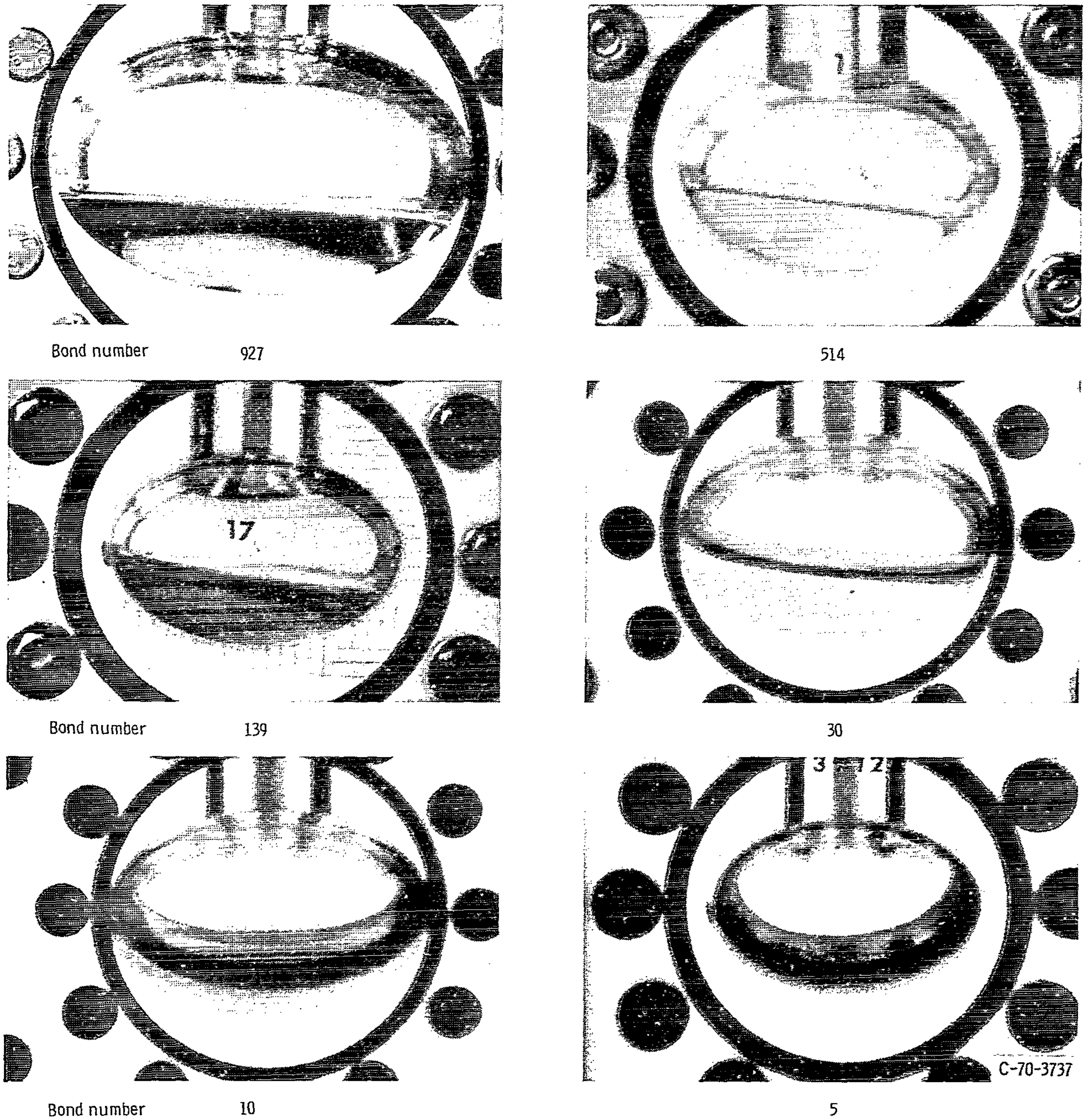
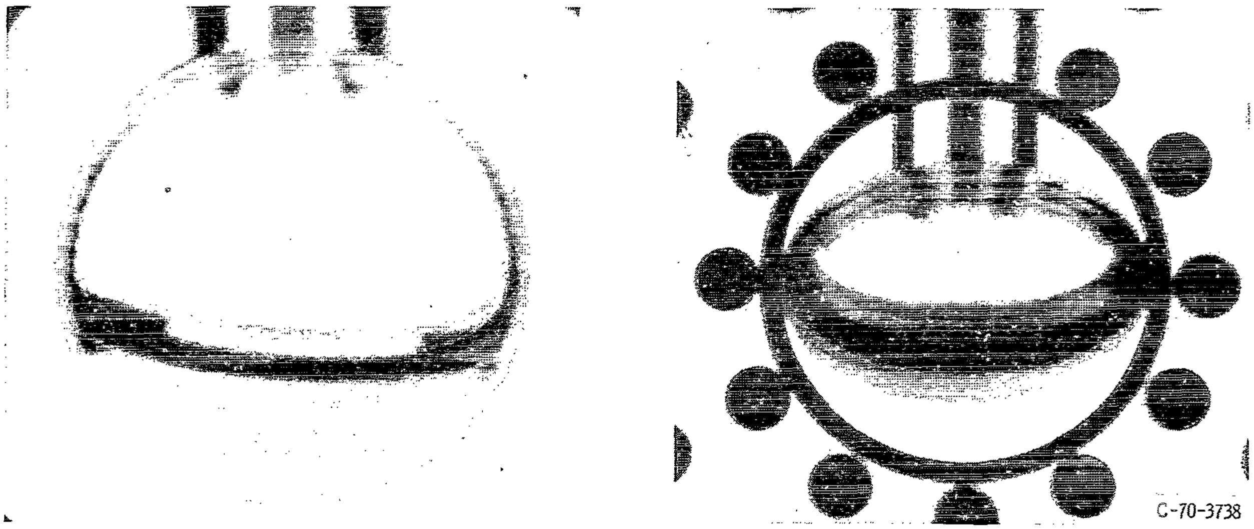


Figure 6. - Fundamental slosh mode at various Bond numbers. Filling, 25 percent; eccentricity, 0.8.



Eccentricity, 0   Eccentricity, 0.8

Figure 7. - Comparison of mode shapes for eccentricities of 0 and 0.8. Filling, 25 percent; Bond number, 10.

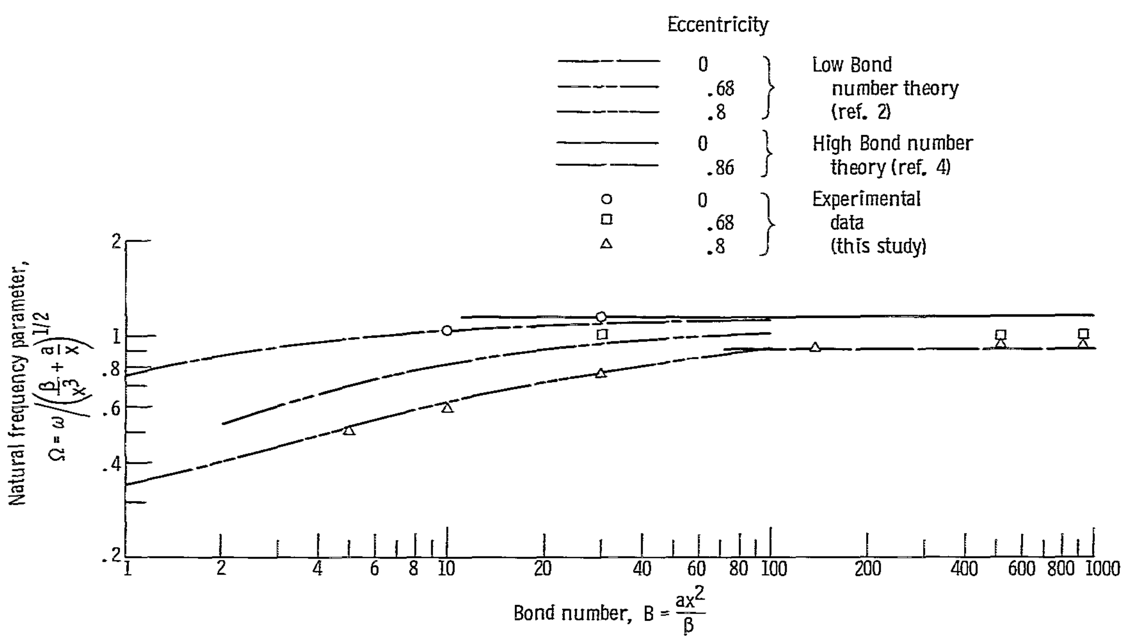


Figure 8. - Natural frequency parameter as function of Bond number. Filling, 25 percent.

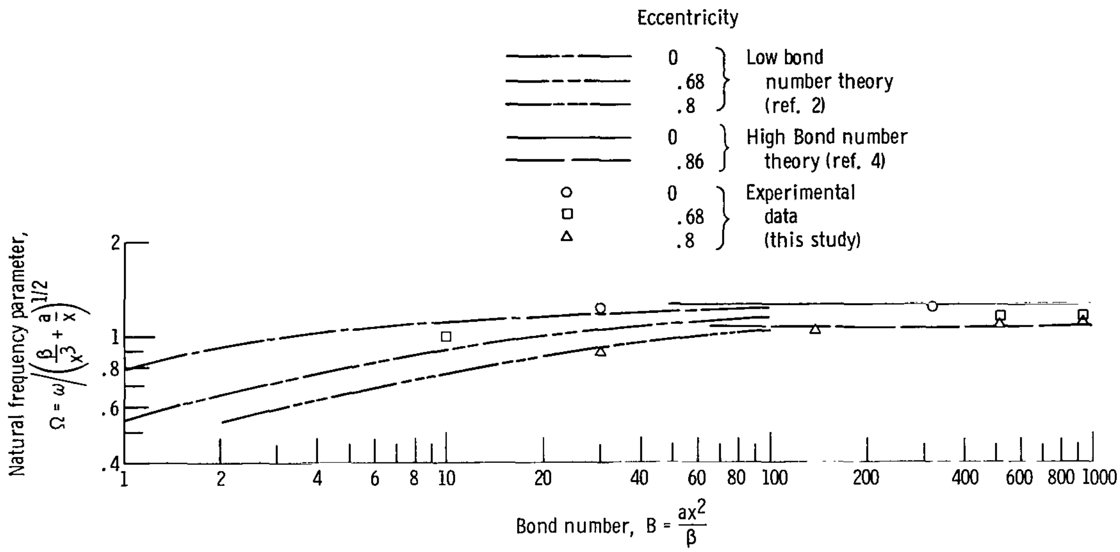


Figure 9. - Natural frequency parameter as function of Bond number. Filling, 50 percent.

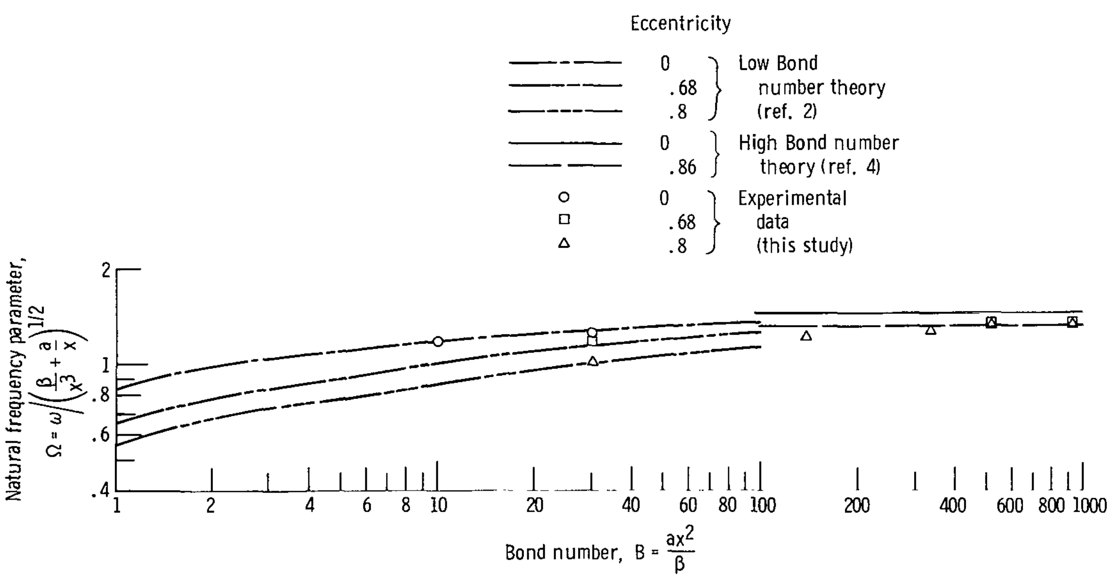


Figure 10. - Natural frequency parameter as function of Bond number. Filling, 75 percent.

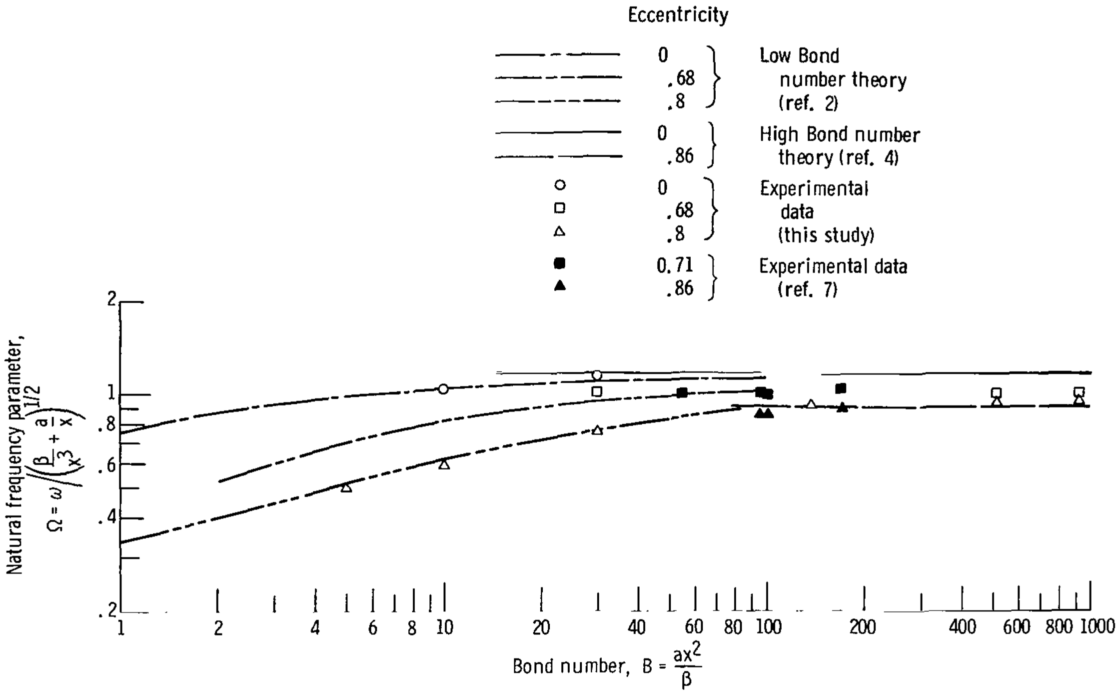


Figure 11. - Comparison of published slosh data. Filling, 25 percent.

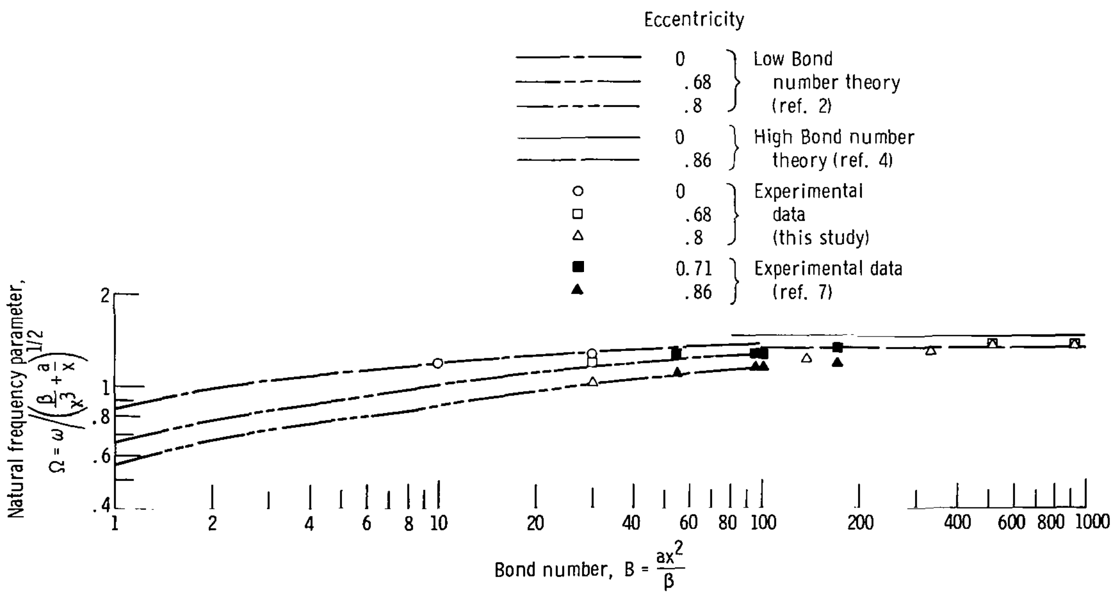
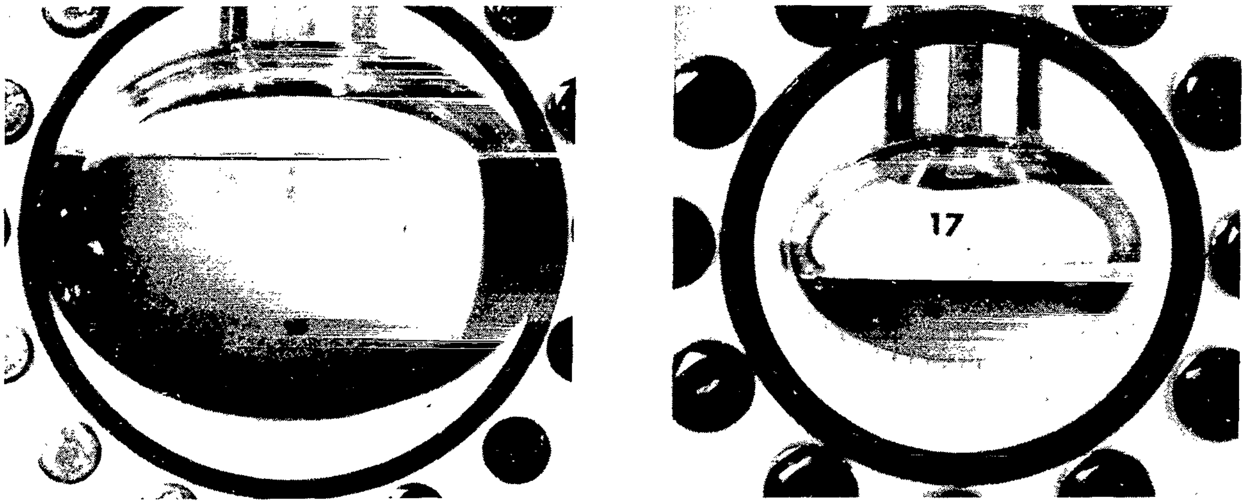
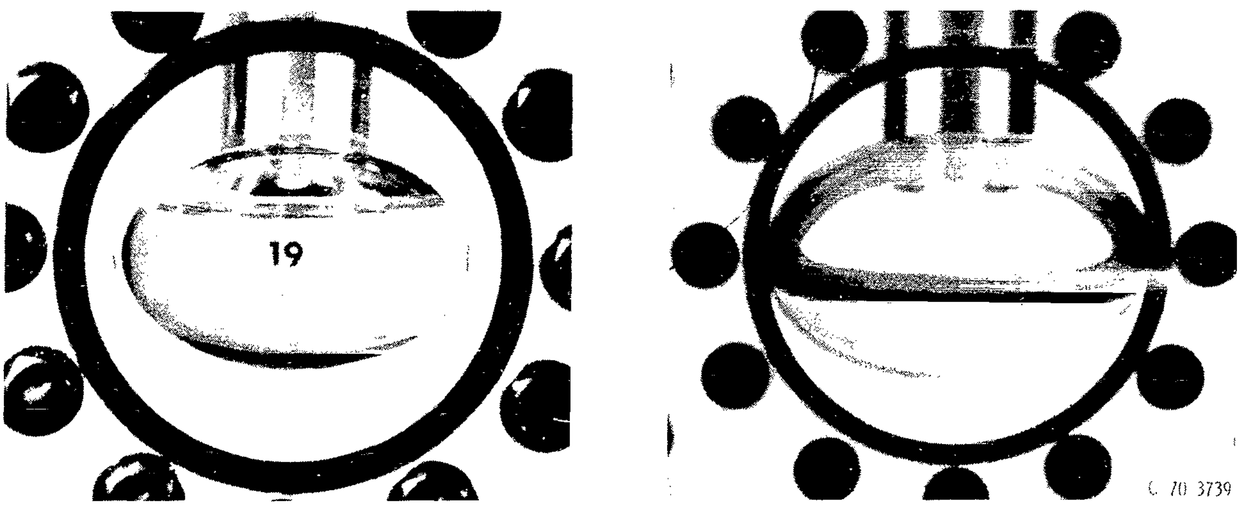


Figure 12. - Comparison of published slosh data. Filling, 75 percent.



Filling, percent	75	25
Bond number	927	139

(a) High Bond number region.



Filling, percent	75	25
Bond number	139	30

(b) Low Bond number region.

Figure 13. - Interface shape for high Bond number region and low Bond number region. Eccentricity, 0.8.

NASA-Langley, 1971 —— 12 E-5842

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON, D.C. 20546

OFFICIAL BUSINESS
PENALTY FOR PRIVATE USE $300

FIRST CLASS MAIL

POSTAGE AND FEES PAID
NATIONAL AERONAUTICS AND
SPACE ADMINISTRATION

02U 001 37 51 3DS 71088 00903
AIR FORCE WEAPONS LABORATORY /WLOL/
KIRTLAND AFB, NEW MEXICO 87117

ATT E. LOU BOWMAN, CHIEF,TECH. LIBRARY

POSTMASTER: If Undeliverable (Section 158 Postal Manual) Do Not Return

"The aeronautical and space activities of the United States shall be conducted so as to contribute . . . to the expansion of human knowledge of phenomena in the atmosphere and space. The Administration shall provide for the widest practicable and appropriate dissemination of information concerning its activities and the results thereof."

—NATIONAL AERONAUTICS AND SPACE ACT OF 1958

NASA SCIENTIFIC AND TECHNICAL PUBLICATIONS

TECHNICAL REPORTS: Scientific and technical information considered important, complete, and a lasting contribution to existing knowledge.

TECHNICAL NOTES: Information less broad in scope but nevertheless of importance as a contribution to existing knowledge.

TECHNICAL MEMORANDUMS: Information receiving limited distribution because of preliminary data, security classification, or other reasons.

CONTRACTOR REPORTS: Scientific and technical information generated under a NASA contract or grant and considered an important contribution to existing knowledge.

TECHNICAL TRANSLATIONS: Information published in a foreign language considered to merit NASA distribution in English.

SPECIAL PUBLICATIONS: Information derived from or of value to NASA activities. Publications include conference proceedings, monographs, data compilations, handbooks, sourcebooks, and special bibliographies.

TECHNOLOGY UTILIZATION PUBLICATIONS: Information on technology used by NASA that may be of particular interest in commercial and other non-aerospace applications. Publications include Tech Briefs, Technology Utilization Reports and Technology Surveys.

Details on the availability of these publications may be obtained from:

SCIENTIFIC AND TECHNICAL INFORMATION OFFICE

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

Washington, D.C. 20546